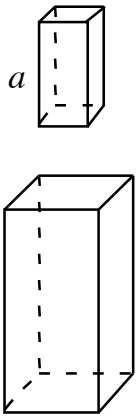
SIMILARITY OF VOLUME

#30

Once you know two figures are similar with a **SCALE FACTOR** or **RATIO OF SIMILARITY** $\frac{a}{b}$, the following proportions for the SMALL (sm) and LARGE (lg) figures (which are enlargements or reductions of each other) are true:



$$\frac{\text{side}_{\text{sm}}}{\text{side}_{\text{lg}}} = \frac{a}{b} \qquad \frac{P_{\text{sm}}}{P_{\text{lg}}} = \frac{a}{b} \qquad \frac{A_{\text{sm}}}{A_{\text{lg}}} = \frac{a^2}{b^2} \qquad \frac{V_{\text{sm}}}{V_{\text{lg}}} = \frac{a^3}{b^3}$$

where P = perimeter, A = area, and V = volume.

In each proportion above, the data from the smaller figure is written on top (in the numerator) to help be consistent with correspondences. When working with volume, the scale factor (ratio of similarity) is cubed. NEVER cube the actual volumes themselves, just the scale factor.

Example

The two rectangular prisms above are similar. Suppose the ratio of their vertical edges is $\frac{3}{7}$.

a. Find the ratio of their surface areas.

b. Find the ratio of their volumes.

c. The volume of the small prism is 30 cubic units. Find the volume of the large prism.

Solutions

a. The ratio of the surface areas is $\frac{A_{\text{sm}}}{A_{\text{lg}}} = \frac{3^2}{7^2} = \frac{9}{49}$.

b. The ratio of the volumes is $\frac{V_{\text{sm}}}{V_{\text{lg}}} = \frac{3^3}{7^3} = \frac{27}{343}$. Use proportions to solve the next part.

c. $\left(\frac{3}{7}\right)^3 = \frac{30}{V}$

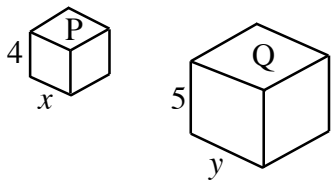
$\frac{27}{343} = \frac{30}{V}$

$27V = 10,290$

$V = \frac{10290}{27} \approx 381.11 \text{ units}^3$

Problems

1. Two rectangular prisms are similar. The smaller, A, has a height of four units while the larger, B, has a height of five units.



   a. What is the scale factor from prism A to prism B?

   b. What is the ratio, small to large, of their volumes?

   c. A third prism, C, is similar to prisms A and B. Prism C's height is ten units. If the volume of prism A is 32 cubic units, what is the volume of prism C?

2. If prism A and prism B have a ratio of similarity of $\frac{2}{1}$, what is the volume of prism B if the volume of prism A is 36 cubic units?

3. If prism A and prism B have a ratio of similarity of $\frac{2}{1}$, what is the volume of prism A if the volume of prism B is 36 cubic units?

4. If prism A and prism B have a ratio of similarity of $\frac{3}{7}$, what is the volume of prism B if the volume of prism A is 83 cubic units?

5. If prism A and prism B have a ratio of similarity of $\frac{3}{7}$, what is the volume of prism A if the volume of prism B is 83 cubic units?

6. If prism A and prism B have a ratio of similarity of $\frac{7}{8}$, what is the volume of prism B if the volume of prism A is 96 cubic units?

7. If prism A and prism B have a ratio of similarity of $\frac{7}{8}$, what is the volume of prism A if the volume of prism B is 96 cubic units?

8. Prism A and prism B are similar. The volume of prism A is 64 cubic units while the volume of prism B is 125 cubic units. What is the ratio of similarity between these two prisms?

9. Prism A and prism B are similar. The volume of prism A is 512 cubic units while the volume of prism B is 125 cubic units. What is the ratio of similarity between these two prisms?

10. Prism A and prism B are similar. The volume of prism A is 8 cubic units while the volume of prism B is approximately 27 cubic units. If the surface area of prism B is 128 square units, what is the surface area of prism A?

11. Prism A and prism B are similar. The volume of prism A is 8 cubic units while the volume of prism B is approximately 27 cubic units. If the surface area of prism A is 128 square units, what is the surface area of prism B?

12. The ratio of the volumes of two similar circular cylinders is $\frac{125}{100}$. What is the ratio of the diameters of their similar bases?

13. The ratio of the volumes of two similar circular cylinders is $\frac{121}{49}$. What is the ratio of the diameters of their similar bases?

14. The surface areas of two cubes are in the ratio of $\frac{25}{49}$. What is the ratio of their volumes?

15. The surface areas of two cubes are in the ratio of $\frac{169}{196}$. What is the ratio of their volumes?

16. The ratio of the weights of two spherical steel balls is $\frac{27}{64}$. What is the ratio of the diameters of the two steel balls?

17. The ratio of the weights of two spherical steel balls is $\frac{64}{8}$. What is the ratio of the diameters of the two steel balls?

Answers

1. a. $\frac{4}{5}$ b. $\frac{64}{125}$ c. $\frac{4}{10} = \frac{2}{5}$, $\frac{2^3}{5^3} = \frac{32}{V} \Rightarrow 8V = 125 \cdot 32 \Rightarrow V = 500$ u^3

2. 4.5 u^3 3. 288 u^3 4. $\approx$ 1054.41 u^3 5. $\approx$ 6.53 u^3

6. $\approx$ 143.3 u^3 7. $\approx$ 64.31 u^3 8. $\frac{4}{5}$ 9. $\frac{8}{5}$

10. $\approx$ 56.89 u^2 11. 288 u^2 12. $\approx \frac{5}{4.64}$ 13. $\approx \frac{4.95}{3.66}$

14. $\frac{125}{343}$ 15. $\frac{2197}{2744}$ 16. $\frac{3}{4}$ 17. $\frac{4}{2} = \frac{2}{1}$